Tax Evasion in an Overlapping Generations Model with Public Investment[*]

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Abstract

The current analysis of the impact of stricter enforcement of tax laws on tax evasion ignores the most policy relevant case where it has effects running from public revenue to public expenditure. The paper shows that the existing results in literature apply to this policy relevant case, but under certain restrictions on the government policy. Using a dynamic general equilibrium model, it shows that the benefits of stricter enforcement are realized immediately but these immediate effects are slightly smaller than the long-run effects derived in the literature. When stricter enforcement is used as a tool to raise revenue for public investment, the positive impact on growth from increased public investment is tempered by a negative general equilibrium effect arising from reduced private capital accumulation.

Keywords: Tax evasion, public investment, dynamic analysis

JEL Classification: H26, H41, C61

1 Introduction

In the current literature, analysis of tax policy, including its effect on tax evasion, typically obviates the need for consideration of the expenditure policy of the government. In normative analysis, this need is obviated by ranking alternative policy

combinations for raising a fixed amount of revenue. In positive analysis, it is implicitly assumed that the resulting change in revenue does not alter the general equilibrium of the economy. The existing analysis of the effects of changes in tax policy on tax evasion is no exception (see Allingham and Sandmo, 1972; Srinivasan, 1973; Cross and Shaw, 1982; and Landskroner, Paroush, and Swary, 1990). This assumption can be justified by allowing government expenditure to enter separably in the agent's utility function or by supposing that the government's revenue and expenditure choices are optimal to begin with.

In practice, however, policy changes are: (1) purposeful and raise revenue for the government, unlike the assumption in normative analysis; and (2) typically aim at increasing productive public spending that alters economy's general equilibrium, contrary to what is assumed in positive analysis. Furthermore, the assumption that the government's revenue and expenditure choices are optimal is a far cry, especially for the developing countries for whom our analysis is particularly relevant.[1] These countries suffer from rampant tax evasion as a large fraction of the population is engaged in informal economy which, as Bearse, Glomm, and Janeba (2000) point out, leads to informational problems, causing difficulty in the assessment of taxable income. As a result, government revenue and expenditure are sub-optimally low. Thus, while the literature considers useful benchmark cases, the most relevant policy case, having effects running from public revenue to public expenditure in a second-best setting (arising from sub-optimal government policy), has fallen into cracks. Moreover, the analysis in the literature is limited to comparative statics in static models.

This paper takes up this missing case, the analysis of which necessarily requires a dynamic model. With productive public expenditure, the effects of policy change are felt over time and the economy spends a significant time on the transition path, away from the steady state. Thus, a complete analysis of tax evasion cannot be conducted in static models. Furthermore, the analysis cannot be restricted to comparative statics, because from the policy perspective, it is far more important to know what happens to the tax evasion in the immediate aftermath of the policy change rather than when the dust has settled.

In the existing literature, the result in Allingham and Sandmo (1972) that the stricter enforcement (an increase in the audit rate or penal tax rate) lowers tax

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[1]Tax evasion is a problem in all economies. Rey (1965) in one of the earliest published estimates found that the evasion of the Italian General Sales Tax was 52.46% of the actual yield. Other similar studies include Gutmann (1977) and Feige (1979). For India, Acharya (1985) estimated that only 53.3% of the total assessable income was declared.

evasion has been extended to more general settings with restrictions placed on the risk-aversion characteristics of the agents.[2] This paper shows that restrictions are required on government policy to further extend these results to revenue-nonneutral policy changes that are purposeful and affect public expenditure. More importantly, given the dynamic general equilibrium setting, we are able to examine not only the long run but also the short run effect of a policy change on tax evasion. While the direction of the effect on tax evasion during transition is same as that across the steady states, the short-run effect is typically smaller than the long-run effect, but only slightly smaller.

The dynamic general equilibrium setting also permits an analysis of the macroeconomic implications of the stricter enforcement of tax laws which is not possible in the existing models of tax evasion. One important result that emerges is that the savings decline with fall in tax evasion.[3] By reducing private capital accumulation, reduced saving has an adverse effect on growth. This effect counteracts the positive effect of stricter enforcement on growth that comes from a greater investment in public capital, made possible by increased revenue collection.

The dynamic effects of public investment on growth have been an important concern in the literature, especially in the context of developing countries. A number of financing options for public investment have been analyzed. Recently, Chatterjee, Sakoulis, and Turnovsky (2003) have studied the effect of tied aid for public investment on economic growth. Chatterjee (2007) analyzes the public versus private provisioning of infrastructure investment (also see Fisher and Turnovsky, 1998). As developing countries have rampant tax evasion, an important tool for increasing public investment is the stricter enforcement of tax laws. In this case, the assessed positive impact on growth from increased public investment needs to be tempered by a negative general equilibrium effect arising from reduced private capital accumulation–a fact that has not been pointed out in the literature.

The paper focuses on the expenditure policy that is geared toward investment in public capital for two reasons. First, the divergence in the return to the public and the private capital is one of the most important example of sub-optimal government policy in developing countries. This is evidenced by the extreme shortage of publicly-provided critical infrastructure in these countries.[4] Second, public investment, being

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[2]Specifically, Cross and Shaw (1982) extend the results to a joint analysis of tax evasion and tax avoidance and Landskroner, Paroush, and Swary (1990) to tax evasion in presence of risky assets. They assume that the agents have decreasing absolute risk aversion.

[3]As the agent cannot insure against being caught evading taxes, prudence (arising from convexity of marginal utility of consumption) requires him to 'self-insure' himself by saving more.

[4]Pohl and Mihaljek (1992) report median and average annual rate of return of 14% and 16%

a very volatile component of government expenditure, responds very sharply to the changes in government revenue. As Roubini and Sachs (1989) note, "in periods of restrictive fiscal policies and fiscal consolidation capital expenditures are the first to be reduced (often drastically)".[5] While econometric estimates of this correlation in the context of tax evasion are hard to come by, a positive correlation between increased tax revenue from stricter tax law enforcement and public infrastructure investment is often taken for granted in the discussions on tax reforms. It is not uncommon to find references to tax evasion and lack of infrastructure in successive paragraphs, if not sentences, implying such a correlation–perhaps, a strong one on the margin. For example, Fuest and Riedel (2009) note, "The provision of public services and infrastructure is a key factor for economic development and growth ... Tax avoidance and tax evasion are widely believed to be important factors limiting revenue mobilisation."[6]

The remaning part of the paper is organized as follows. The model is outlined in section 2 and solved in section 3. Section 4 analytically characterizes the effects of a policy change on tax evasion and macroeconomic variables for a simplified case. Numerical simulations of section 5 extend results of section 4 to more general cases. Section 6 concludes.

2 The Model

The paper considers consider a two-period overlapping generations economy with constant population. The population of each generation has measure 1 with each individual having a zero measure. Each agent is endowed with one unit of labor when young. When old he retires and lives on the income from derived from the capital that is accumulated when young. The production requires labor, private capital, and public capital. The government finances public capital by levying taxes which agents can evade. It fights tax evasion by auditing tax returns and levying a penal tax on

respectively for 1,015 World Bank projects undertaken in developing countries—much higher than for private capital. Easterly (1999) summarizes evidence showing that the return to public investment in developing countries (in physical infrastructure) may actually be even higher (19%-29%).

[5] This agrees with the findings in the *World Development Report* (1988) that, in the face of fiscal tightening, the public investment fell far more sharply (35%) than other current expenditures such as wages (10%). Hicks (1991) comes up with corresponding estimates of 27.8% and 7.2%.

[6] Similarly, a *Xinhua* news item in 2006 stated, "China's tax revenues will increase by between 700 billion yuan and 800 billion yuan (89 billion to 101 billion U.S. dollars) this year, mainly due to stricter taxation enforcement ... The tax revenue has played an important role in developing the country's infrastructure. (Strict taxation enforcement to push China's tax revenue to new high, *Xinhua* 16 October 2006.)

the evaded income.

2.1   Preferences and Utility Maximization

The agents have time-additive separable (von Neumann-Morgenstern) utility function. Each individual derives utility from consumption in both periods. No utility is derived from leisure, and hence, the young supply their entire endowment of labor to the firms. The agents do not have bequest motive. There are standard assumptions on the utility function. The per period utility function $u(.)$ is strictly increasing, strictly concave, twice continuously differentiable, and satisfies Inada conditions.

The representative agent of generation t, when young, earns wage income w_t and, in addition, also receives transfers j_t from the government. Since, he supplies one unit of labor, w_t is also the wage rate in period t. In period $t + 1$, when old, he derives income from the capital accumulated in period t. The government levies a tax at the rate $\tau_{i,t}$ on the labor income earned in period t which agents can evade. It audits a fraction p of the returns. When a return is audited, tax evasion is detected with probability 1. Thus, in the model, the audit rate coincides the probability of being caught. When caught, the agents pay a penal tax on the unreported labor income, in period t itself, at a higher rate $\tau_{i,t}^p$.

In this model, therefore, besides the standard consumption-saving decision, an agent of generation t also takes a decision on the fraction of the labor income on which to evades taxes. He cannot diversify away the risk arising from tax evasion; although, at the time of making this decision, he knows the probability of being caught and the penal tax rate $\tau_{i,t}^p$. Based on this information, an agent of generation t, when young, decides the fraction of labor income (x_t) to hide and the saving (s_t) for the next period. After taking this decision, he learns in the same period if he has been caught. If caught, he pays penal taxes from his savings. The model, therefore, embeds the framework used by Allingham and Sandmo (1972) to study tax evasion in a dynamic general equilibrium model.

The representative agent of generation t, thus, accumulates $k_{1,t+1}$ amount of capital if he is not caught and $k_{2,t+1}$ otherwise, and he solves the following optimization problem to maximize his lifetime utility:

$$\max_{c_{y,t},s_t,x_t,c^1_{o,t+1},c^2_{o,t+1}} u(c_{y,t}) + \beta\left(1 - p\right) u(c^1_{o,t+1}) + \beta p u\left(c^2_{o,t+1}\right) \tag{1}$$

subject to

$$c_{y,t} + s_t \leq [x_t + (1 - x_t)(1 - \tau_{i,t})] w_t + j_t, \tag{2}$$

$$c_{o,t+1}^1 \leq [r_{t+1} + (1 - \delta_k)] k_{1,t+1}, \tag{3}$$

$$c_{o,t+1}^2 \leq [r_{t+1} + (1 - \delta_k)] k_{2,t+1}, \tag{4}$$

$$k_{1,t+1} = s_t, \tag{5}$$

$$k_{2,t+2} = s_t - \tau_{i,t}^p x_t w_t, \tag{6}$$

$$0 \leq x_t \leq 1,$$

where $c_{y,t}$, is the consumption of an agent of generation t when he is young, $c_{o,t+1}^1$ is his consumption when old (i.e., in period $t+1$) if he is not caught evading taxes, $c_{o,t+1}^2$ is the consumption when old if he is caught evading taxes, and r_{t+1} is the (gross) return on capital from period t to period $t+1$. As the utility function is strictly increasing in consumption in each period, all budget constraints hold with equality in equilibrium.

The dynamics of the model is very simple. All young are born equal and, being homogeneous, solve the same problem. They decide to evade tax on the same fraction of income and save the same amount for the next period. However, when old they get split into two types: a fraction p who were caught evading taxes have lower per capita capital and the others not caught have higher capital per capita. Thus, at any point in time there are only three types of agents in the economy–current young and two types of old– and the private capital is owned by the two types of old. The aggregate capital is the weighted average of the capital of these two types.

2.2 Technology and Profit Maximization

On the production side, following Futagami, Morita, and Shibata (1993), public capital (G) augments the productivity of the firms. The technology is Cobb-Douglas with output given by

$$F(K, G, L) = AG^\theta K^\alpha L^{1-\alpha} \equiv Y, \tag{7}$$

where $\alpha > 0$, $\theta > 0$ and $\alpha + \theta < 1$, and G, K, L, and Y are economy-wide aggregates. Note that due to decreasing returns in accumulable factors, there is no long-run growth in the economy. The output per person of the generation supplying labor is

$$y = f(k, G) = AG^\theta (K/L)^\alpha = AG^\theta k^\alpha. \tag{8}$$

The firm hires capital and labor to maximizes profit and its optimization problem is:

$$\max_{K_t, L_t} \quad \pi_t = F(K_t, G_t, L_t) - r_t K_t - w_t L_t, \tag{9}$$

where it takes the current stock of public capital, G_t, as given.

2.3 The Government's Budget Constraint

The government receives revenue both from the statutory tax on labor income and the penal tax on evaded income. Thus, the government's revenue, R_t, is

$$R_t = \tau_{i,t} \left(1 - X_t\right) W_t + p \tau_{i,t}^p X_t W_t, \tag{10a}$$

where W_t is the aggregate wage income of the agents of generation t in period t and X_t is the average fraction of income not reported by them. This revenue is either invested in public capital or rebated back to the current young from whom it is collected. The government's budget constraint for period t is, therefore, given by

$$R_t = G_{t+1} - (1 - \delta_G) G_t + J_t, \tag{10b}$$

where δ_G is the rate of depreciation of public capital, J_t is the total transfer made to generation t in period t, and G_{t+1} is the new stock of the public capital that enters the production function of the firms in period $t+1$.

Having laid out the model, we now define a competitive equilibrium for this economy.

Definition. *A competitive equilibrium for this economy is a sequence $\{c_{y,t},\ c_{o,t+1}^1,\ c_{o,t+1}^2,\ x_t,\ s_t,\ k_{1,t+1},\ k_{2,t+1},\ r_t,\ w_t,\ K_t,\ L_t,\ G_t,\ j_t,\ J_t,\ \tau_{i,t},\ \tau_{i,t}^p\}$ such that:*

1. *For every t, given $\{r_{t+1},\ w_t,\ j_t,\ \tau_{i,t},\ \tau_{i,t}^p\}$, $\{c_{y,t},\ c_{o,t+1}^1,\ c_{o,t+1}^2,\ x_t,\ s_t\}$ solves the optimization problem (1) for the agents of generation t.*

2. *For every t, given $\{r_t,\ w_t,\ G_t\}$, $\{K_t,\ L_t\}$ maximizes the profit of the firms in (9).*

3. *For every t, given $\{c_{y,t},\ c_{o,t+1}^1,\ c_{o,t+1}^2,\ x_t,\ s_t,\ k_{1,t+1},\ k_{2,t+1},\ r_{t+1},\ w_t,\ K_t,\ L_t, G_t\}$, government policy $\{G_{t+1},\ J_t,\ \tau_{i,t},\ \tau_{i,t}^p\}$ satisfies its budget constraint (10a-10b).*

7

4. *All markets clear and the aggregate and the individual variables are consistent for all t. Specifically the markets for capital, labor, and output clear:*[7]

$$C_{y,t} = c_{y,t}, \qquad C_{o,t-1}^1 = (1-p)c_{o,t-1}^1, \qquad C_{o,t-1}^2 = pc_{o,t-1}^2,$$

$$K_{t+1} = (1-p)k_{1,t+1} + pk_{2,t+1} = S_t - p\tau_{i,t}^p X_t W_t, \tag{11}$$

$$L_t = 1, \quad J_t = j_t, \quad S_t = s_t, \quad X_t = x_t, \quad W_t = w_t,$$

$$C_{y,t} + C_{o,t-1}^1 + C_{o,t-1}^2 + (K_{t+1} - (1-\delta_k)K_t) + (G_{t+1} - (1-\delta_G)G_t) = Y_t.$$

3 Solving the Model

To solve for the competitive equilibrium of the model, we begin with the firms' problem (9), which gives the following first-order conditions:

$$K_t \quad : \quad r_t = F_{K,t} = \alpha \frac{Y_t}{K_t}, \tag{12}$$

$$L_t \quad : \quad w_t = F_{L,t} = (1-\alpha)\frac{Y_t}{L_t} = (1-\alpha)Y_t. \tag{13}$$

Turning to the utility maximization by agents, the Kuhn-Tucker conditions for their problem (1) for the choices of s_t and x_t are

$$s_t \quad : \quad u'(c_{y,t}) = \beta \left[(1-p)u'\left(c_{o,t+1}^1\right) + pu'\left(c_{o,t+1}^2\right) \right] \left[F_{K,t+1} + (1-\delta_k) \right], \tag{14}$$

$$x_t \quad : \quad \tau_{i,t}u'(c_{y,t}) \leq \beta p\tau_{i,t}^p u'\left(c_{o,t+1}^2\right) \left[F_{K,t+1} + (1-\delta_k) \right] \quad \text{if} \quad x_t = 0, \tag{15a}$$

$$\quad : \quad \tau_{i,t}u'(c_{y,t}) = \beta p\tau_{i,t}^p u'\left(c_{o,t+1}^2\right) \left[F_{K,t+1} + (1-\delta_k) \right] \quad \text{if} \quad 0 \leq x_t \leq 1, \tag{15b}$$

$$\quad : \quad \tau_{i,t}u'(c_{y,t}) \geq \beta p\tau_{i,t}^p u'\left(c_{o,t+1}^2\right) \left[F_{K,t+1} + (1-\delta_k) \right] \quad \text{if} \quad x_t = 1. \tag{15c}$$

In the first order condition for x_t, the left hand side is the marginal benefit of evading taxes and the right hand side is the marginal cost. Thus, for a corner solution with no tax evasion, i.e., $x_t = 0$, the marginal cost is higher than the marginal benefit.

Finally, from the market clearing condition for private capital (11) and the government's budget constraint (10a-10b), the aggregate stocks of the private and the public capital for the next period are given by

$$K_{t+1} \quad = \quad s_t - p\tau_{i,t}^p x_t w_t, \tag{16}$$

$$G_{t+1} \quad = \quad \left[\tau_{i,t} - \left(\tau_{i,t} - p\tau_{i,t}^p \right) x_t \right] w_t + (1-\delta_G)G_t - j_t, \tag{17}$$

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[7]Note that, for clarity, the variables for individuals are denoted by small letters and economy wide variables by corresponding capital letters.

where the aggregate variables have been replaced by individual variables using the equilibrium conditions.

For solving the model, note that given the government policy $\left\{p, \tau_{i,t}^{p}, \tau_{i,t}, j_{t}\right\}$ and the initial capital stocks (K_t, G_t), (14-17) are 4 equations in 4 unknowns, namely, current decisions of the agents, s_t and x_t, and the stock of private and public capital, K_{t+1} and G_{t+1}. Given (K_0, G_0), it is, therefore, possible to recursively solve the model. If the solution for x_t is interior, one would use equation (15b). While typically this will be true in practice, occasionally there may be a corner solution and this fact must be taken into account while solving the system. In particular, there is a corner solution under the following condition:

Proposition 1. *If the expected penal rate is at least as high as the statutory tax rate, i.e., if $p\tau_{i,t}^{p} \geq \tau_{i,t}$, there exists a unique equilibrium with no tax evasion.*

Allingham and Sandmo (1972), Yitzhaki (1974) and Landskroner et al. (1990) establish this result for the static models in a partial equilibrium setting. The result follows from the risk-aversion of the agents and is quite intuitive.[8] Proposition 1 generalizes the result to a dynamic general equilibrium model. The reason this result hold in this more general setting is easy to see. The key fact is that the uncertainty arising from tax evasion is resolved after the agent has made the consumption-saving decision. Given the amount the agent has decided to save, the agent can choose to either pay evaded taxes up-front or take the risk of being caught evading taxes. The trade-off he faces is exactly the same as in the static model. The only difference is that, instead of affecting consumption out of an exogenously given income, evading taxes now affects future consumption out of his saving which is endogenous.

As our purpose is to analyze tax evasion, in what follows, it will be assumed that $p\tau_{i,t}^{p} < \tau_{i,t}$.

4   Analytical Characterization in a Simplified Model

This section provides a detailed analytical characterization of tax evasion in a simplified model with logarithmic utility function, i.e.,

$$u(c) = \log(c), \tag{18}$$

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[8]When $p\tau_{i,t}^{p} = \tau_{i,t}$, the expected tax liability is same whether the agent evades or pays taxes, but due to risk aversion or the concavity of the utility function, certainty equivalent of the expected loss, if the agent chooses to evade taxes, is larger than the certain loss from paying taxes. For $p\tau_{i,t}^{p}$ larger than $\tau_{i,t}$ the trade-off only gets worse.

and complete depreciation of both types of capital.[9] As we will see, this simplified set up serves as a useful benchmark for understanding the effect of changes in government policy on tax evasion. The results for the more general model, that relaxes the simplifying assumptions turn out to be very similar as shown by the numerical simulations of the next section.

4.1 Analytical Solution

For log utility, Kuhn-Tucker conditions for utility maximization $(14-15)$ become

$$s_t \quad : \quad \frac{1}{c_{y,t}} = \frac{\beta(1-p)}{s_{t+1}} + \frac{\beta p}{s_{t+1} - \tau_{i,t}^p x_t w_t}, \tag{19}$$

$$x_t \quad : \quad \frac{\tau_{i,t}}{c_{y,t}} \leq \frac{\beta p \tau_{i,t}^p}{s_{t+1} - \tau_{i,t}^p x_t w_t} \quad \text{if } x_t = 0, \tag{20a}$$

$$: \quad \frac{\tau_{i,t}}{c_{y,t}} = \frac{\beta p \tau_{i,t}^p}{s_{t+1} - \tau_{i,t}^p x_t w_t} \quad \text{if } 0 \leq x_t \leq 1, \tag{20b}$$

$$: \quad \frac{\tau_{i,t}}{c_{y,t}} \geq \frac{\beta p \tau_{i,t}^p}{s_{t+1} - \tau_{i,t}^p x_t w_t} \quad \text{if } x_t = 1. \tag{20c}$$

Imposing equilibrium, one can solve (19) and (20b) for the equilibrium *ex ante* individual savings function, $\tilde{s}^t$, and individual 'tax evasion' function, x^t, for the case when there is an interior solution.[10] These functions are

$$\frac{S_t}{Y_t} = \frac{s_t}{y_t} = \tilde{s}^t(K_t, G_t) = (1-p) \frac{\beta}{1+\beta} \frac{\tau_{i,t}^p}{\tau_{i,t}^p - \tau_{i,t}} \left[(1-\tau_{i,t})(1-\alpha) + \frac{j_t}{y_t} \right], \tag{21}$$

$$X_t = x_t = x^t(K_t, G_t) = \frac{1}{1-\alpha} \frac{\beta}{1+\beta} \left[\frac{1}{\tau_{i,t}^p} - \frac{p}{\tau_{i,t}} \right] \frac{\left[(1-\tau_{i,t})(1-\alpha) + \frac{j_t}{y_t} \right]}{\left[1 - \frac{\tau_{i,t}}{\tau_{i,t}^p} \right]}, \tag{22}$$

where y_t on the right hand side of (21-22) is a function of (K_t, G_t) and superscript t denotes the dependence of the decision rules or functions on the government policy at time t. Since population size of each generation is normalized to 1 and only the current young, who are homogeneous *ex ante*, save and evade taxes, these functions also represent economy wide averages as well as aggregates.

The solution in $(21-22)$ can be used in $(16-17)$ to solve for the next period

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[9]The assumption of complete depreciation of private capital can be relaxed without affecting the results. The relaxation of the assumption of complete depreciation of public capital will only affect the results concerning the dynamics of the public capital.

[10]If the agent is caught evading taxes, *ex post* savings is lesser by the amount of penal taxes paid by him.

values of (aggregate) state variables K and G

$$K_{t+1} = K^t(K_t, G_t) = \frac{\beta}{1+\beta} \left[\frac{\tau_{i,t}^p}{\tau_{i,t}} p^2 + \frac{\tau_{i,t}^p}{\tau_{i,t}^p - \tau_{i,t}} (1-p)^2 \right] \left[(1-\tau_{i,t})W_t + J_t \right], \quad (23)$$

$$G_{t+1} = G^t(K_t, G_t) = \left[\tau_{i,t} + \frac{\beta\xi_t}{1+\beta}(1-\tau_{i,t}) \right] W_t - \left[1 - \frac{\beta\xi_t}{1+\beta} \right] J_t, \quad (24)$$

where

$$\xi_t = -\frac{\tau_{i,t}}{\tau_{i,t}^p - \tau_{i,t}} \left(1 - p\frac{\tau_{i,t}^p}{\tau_{i,t}} \right)^2, \quad \text{with} \quad \frac{\partial \xi_t}{\partial p} > 0, \quad \frac{\partial \xi_t}{\partial \tau_i^p} > 0, \quad \text{and} \quad \frac{\partial \xi_t}{\partial \tau_i} < 0.$$

Equations $(23 - 24)$ describe the equilibrium dynamics of the economy given (K_0, G_0).

4.1.1 Analytical Solution for Policy Analysis

In this paper, we are interested in the effect of change in the audit rate and the penal tax rate on tax evasion and macroeconomic variables. This analysis of government policy is most transparent for a one time change in otherwise time-invariant government policies $\{p, \tau_i^p, \tau_i\}$. However, as a change in p or τ_i^p also changes the revenue of the government, a complete specification of the policy change has to specify which item of expenditure adjusts to satisfy its budget constraint. Since we will deal with time-invariant policies, it is equivalent and more natural to specify which of the two items of expenditure, public investment or transfers, is invariant to the policy change.

The empirical evidence cited earlier suggests that public investment responds much more strongly to the variation in government revenue than other components of government expenditure. In accordance with this evidence, we will consider the following two, naturally extreme, cases where transfers remain unchanged in an appropriate sense whereas public investment adjusts:

Case 1 *The transfers remain constant as fraction of GDP when government policy changes, and the change in government revenue (as fraction of GDP) goes entirely towards the increase in public investment. Formally, $\zeta_Y \equiv J_t/Y_t$ is exogenous and invariant to the change in policy.*

Case 2 *The transfers remain constant as fraction of government revenue when government policy changes, and the change in government revenue (as fraction of GDP) changes both the public investment and the transfers proportionally. Formally, $\zeta_R \equiv J_t/R_t$ is exogenous and invariant to the change in policy.*

11

For Case 1, the equations $(21-24)$ characterizing the decision rules of the agents and the macroeconomic dynamics of the economy simplify to

$$\frac{S_t}{Y_t} = \frac{s_t}{y_t} = \tilde{s}(K_t, G_t) = (1-p)\frac{\beta}{1+\beta}\frac{\tau_i^p}{\tau_i^p - \tau_i}\left[(1-\tau_i)(1-\alpha) + \zeta_Y\right], \qquad (21a)$$

$$X_t = x_t = x(K_t, G_t) = \frac{1}{1-\alpha}\frac{\beta}{1+\beta}\left[\frac{1}{\tau_i^p} - \frac{p}{\tau_i}\right]\frac{\left[(1-\tau_{i,t})(1-\alpha) + \zeta_Y\right]}{\left[1 - \frac{\tau_i}{\tau_i^p}\right]}, \quad (22a)$$

$$K_{t+1} = K(K_t, G_t) = \frac{\beta}{1+\beta}\left[\frac{\tau_i^p}{\tau_i}p^2 + \frac{\tau_i^p}{\tau_i^p - \tau_i}(1-p)^2\right]\left[(1-\tau_i) + \frac{\zeta_Y}{1-\alpha}\right]W_t \quad (23a)$$

$$G_{t+1} = G(K_t, G_t) = \left[\tau_i - \frac{\zeta_Y}{1-\alpha} + \frac{\beta\xi}{1+\beta}\left\{1 - \tau_i + \frac{\zeta_Y}{1-\alpha}\right\}\right]W_t, \qquad (24a)$$

whereas for Case 2, the analytical solution of the model in $(21-24)$ simplifies to

$$\tilde{s}_{t+1} = \tilde{s}(K_t, G_t) = \frac{(1-p)\beta}{1+\beta(1-\xi\zeta_R)}\frac{\tau_i^p}{\tau_i^p - \tau_i}\left[(1-\tau_i) + \tau_i\zeta_R\right](1-\alpha) \quad (21b)$$

$$x_t = x(K_t, G_t) = \frac{\beta}{1+\beta(1-\xi\zeta_R)}\left[\frac{1}{\tau_i^p} - \frac{p}{\tau_i}\right]\frac{\left[(1-\tau_i) + \tau_i\zeta_R\right]}{\left[1 - \frac{\tau_i}{\tau_i^p}\right]}, \qquad (22b)$$

$$K_{t+1} = K(K_t, G_t) = \left[\frac{\tau_i^p p^2}{\tau_i} + \frac{\tau_i^p(1-p)^2}{\tau_i^p - \tau_i}\right]\frac{\beta\left[(1-\tau_i) + \tau_i\zeta_R\right]W_t}{1+\beta(1-\xi\zeta_R)}, \qquad (23b)$$

$$G_{t+1} = G(K_t, G_t) = (1-\zeta_R)\frac{\left[(1+\beta)\tau_i + \beta\xi(1-\tau_i)\right]}{1+\beta(1-\xi\zeta_R)}W_t. \qquad (24b)$$

It may be noted that in this simplified model, when government policies are time-invariant, saving and tax evasion depend only on the policy of the government and not on the aggregate state variables (K_t, G_t) of the economy. This is evident from inspection of equations $(21-22)$ as well as $(21a-22a)$ and $(21b-22b)$.

4.2 Steady State and Comparative Statics

The remark at end of the previous section implies that the steady state tax evasion and saving does not depend on the stock of private and the public capital in the steady state (K^*, G^*). Thus, without prejudice, we defer the computation of steady state and begin with the comparative static analysis of tax evasion and saving.

4.2.1 Tax Evasion and Saving with No Transfers

It is instructive to begin the analysis with the case in which there are no transfers so that $j^* = 0$. This is not only the simplest case but it is also a special case of both

Case 1 and Case 2. It allows us to identify different mechanisms, operating in the model, in a clear and transparent way. Moreover, the extension to Case 1 and 2 is straightforward and clearly brings out the role of assumptions involved in the two cases.[11]

We begin with the effect of government policy on tax evasion that is summarized in the following proposition:

Proposition 2. *When* $j^* = 0$,

$$\frac{\partial x^*}{\partial p} < 0, \quad \text{and} \quad \frac{\partial x^*}{\partial \tau_i^p} < 0.$$

Proposition 2 shows that an increase in p or τ_i^p decreases the steady state rate of tax evasion. Thus, it extends the existing result in literature to a dynamic general equilibrium setting, albeit to a special case. As in those models, an increase in either p or τ^p makes tax evasion costlier and leads to a negative substitution effect (as defined in Allingham and Sandmo, 1972) on x^*. Besides making tax evasion costlier, stricter enforcement also reduces the agent's post-tax income by increasing the effective rate of taxation. However, this does not affect tax evasion because with CRRA preferences (of which log-preferences is a special case) there is no income effect on tax evasion. Closely tied to the effect of stricter enforcement on tax evasion is its effect on saving.

Proposition 3. *When* $j^* = 0$, *the responses of saving and tax evasion to a change in either* p *or* τ_i^p *are positively correlated, i.e.,*

$$\frac{\partial \tilde{s}^*}{\partial x^*} > 0.$$

This result is quite intuitive. Heuristically, the argument is as follows: The negative substitution effect of an increase in p or τ_i^p causes the agent to reduce tax evasion. Since, the marginal utility is convex in consumption with CRRA preferences, the agents are prudent in sense of Kimball (1990). When they increase tax evasion, this prudence motivates the agents to also save a larger proportion of their income. Thus, due to the substitution effect on tax evasion, saving and tax evasion move together in the same direction.

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[11]This case is also interesting for another reason as it is analytically equivalent to the cases where increased government revenues from stricter tax law enforcement are used for unproductive public spending or for a public consumption good that enters separably in the utility function. Thus, while in the paper we concentrate on productive public infrastructure investment, the results also hold for these alternative uses of additional tax revenues. Of course, results concerning public capital are irrelevant in these cases.

By virtue of the simplicity of the current setup, this intuition is easily verified by the analytical solution. After some simple algebra, the proportion of income saved by the agent can be written as

$$\frac{s^*}{w^* \left(1 - (1 - x^*) \tau_i\right)} = \frac{\beta}{1 + \beta} + \frac{1}{1 + \beta} \frac{x^* \tau_i}{1 - \tau_i + x^* \tau_i}. \tag{25}$$

The first term on the right hand side captures the usual consumption-smoothing motive for saving: as a log-utility consumer, the agent saves a fraction $\beta / (1 + \beta)$ of his post-tax income. The second, positive term on the right hand side captures the prudential motive for saving: depending on how much risk he is endogenously taking (which depends positively on x^*) prudence also demands him to save somewhat more.

Proposition 2 and 3 readily lead to the following corollary summarizing the effect of government policy on saving:

Corollary 1. *When $j^* = 0$,*

$$\frac{\partial \tilde{s}^*}{\partial p} < 0, \quad \text{and} \quad \frac{\partial \tilde{s}^*}{\partial \tau_i^p} < 0.$$

Thus, indeed, stricter tax law enforcement leads to both reduced tax evasion and saving as our intuitive discussion above suggests.

4.2.2 Capital Accumulation with No Transfers

Recall, the model has no long-run growth. Thus, the steady state is characterized by unchanging aggregate stocks of the private and the public capital as well as the output. We begin by characterizing the effect of government policy on capital-output ratios in the economy. For this note that using either $(22a - 23a)$ or $(22b - 23b)$ one gets

$$\frac{K^*}{Y^*} = \frac{\beta}{1 + \beta} \left[\frac{\tau_i^p}{\tau_i} p^2 + \frac{\tau_i^p}{\tau_i^p - \tau_i} (1 - p)^2\right] (1 - \tau_i)(1 - \alpha), \tag{26}$$

$$\frac{G^*}{Y^*} = \left[\tau_i + \frac{\beta \xi}{1 + \beta} (1 - \tau_i)\right] (1 - \alpha), \tag{27}$$

from which it follows that:

Proposition 4. *When $j^* = 0$,*

$$\frac{\partial}{\partial p} \left(\frac{K^*}{Y^*}\right) < 0, \quad \frac{\partial}{\partial \tau_i^p} \left(\frac{K^*}{Y^*}\right) < 0, \quad \text{and} \quad \frac{\partial}{\partial p} \left(\frac{G^*}{Y^*}\right) > 0, \quad \frac{\partial}{\partial \tau_i^p} \left(\frac{G^*}{Y^*}\right) > 0.$$

Thus, a stricter enforcement not only reduces saving (Corollary 1) but also reduces the private-capital-to-output ratio in the economy. On the other hand, the ratio of public capital to output rises. These two outcomes have opposing effects on the output of the economy as one can see by writing the output of the economy as

$$Y^* = A \left[\left(\frac{G^*}{Y^*} \right)^\theta \left(\frac{K^*}{Y^*} \right)^\alpha \right]^{\frac{1}{1-\theta-\alpha}}.$$

(28)

In infrastructure-starved developing countries, one important purpose of the policy of stricter tax law enforcement would be to increase public investment. In contrast to the effect of public investment on growth when it is financed in other ways (Chatterjee, Sakoulis, and Turnovsky, 2003 and Chatterjee, 2007), in this case, its direct positive effect on growth needs to be tempered by this negative general equilibrium effect arising from reduced private capital accumulation.[12] For developing countries with scarce infrastructure, while it is unlikely that this negative effect will be the dominant one, but in light of widespread tax evasion, it would certainly be important. Yet, it is also quite likely that stricter enforcement will be welfare improving as the distortions from both tax evasion and low public investment will fall.

A vast empirical literature on public capital-growth link has been spawned by a series of papers by Aschauer in 1989 (e.g., see Aschauer, 1989a, 1989b). The literature, however, has been inconclusive with estimates of the elasticity of output with respect to public capital ranging from 0 to as high as .58.[13] While more recent work has reduced this range, a satisfactory resolution of this issue requires structural econometric estimation based on a well-specified general equilibrium model. This paper suggests that, in the context of developing countries, the general equilibrium model underlying structural econometric estimation should incorporate tax evasion decision of the agents. This is especially important if one is interested not just in the direct effect of public capital on growth but on the overall general equilibrium

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[12]While here the effects are on the level of output, in a model with long-run growth, the effect will be on the growth *rate* of the output.

[13]The earlier high estimates of productivity of public capital (Aschauer, 1989a; Holtz-Eakin, 1988; Munnel 1990) have been found to be severely distorted by measurement and econometric problems. Estimation using the concept of core infrastructure and employing first differencing to eliminate common time trends lead to much lower, often negative, and statistically insignificant estimates (Hulten and Schwab 1991). Using more advanced econometric techniques (Generalized Method of Moments and instrumental variables) Ai and Cassou (1995) find statistically significant estimates of the elasticity of output with respect to public capital in the range of $.1 - .2$ which is in the middle of the extremes (also see Lynde and Richmond, 1993). The more recent literature has looked at the bias resulting from aggregating across sectors (Connolly and Fox, 2004) and geographic regions (Haughwout, 2002).

effect of stricter tax enforcement on growth where the direct, positive effect needs to tempered by the negative, indirect effect.

4.2.3 The Effect of Tax Evasion on Macroeconomic Variables

The results of the previous section show that an economy with higher tax evasion has higher ex ante savings and private capital accumulation. These results follow from an increase in precautionary saving that agents undertake to insure themselves against the likelihood of getting caught. This clearly implies that the aggregate saving will rise with tax evasion, but, due to two conflicting effects, it is not immediately apparent that aggregate private capital stock will rise as well. After all, the agents caught evading have less capital whereas those lucky have higher. The reason the second effect is unambiguously stronger is that the agents increases saving more than his expected penal tax payment to partially respond to exceptionally low consumption in the eventuality of being caught. Therefore, not only does the aggregate saving but private capital accumulation rises as well.

4.2.4 Analysis with Transfers

Although we have analyzed the effect of stricter tax law enforcement for the special case with no transfers, our primary interest is in the two policy-relevant cases–namely, Case 1 and Case 2–that were outlined in section 4.1. As the no-transfer case is a special case of both cases, the previous results are applicable to these cases, at least to some extent. This section examines the applicability of the previous results to Case 1 and Case 2 beyond the special case with no transfers.

For Case 1 this is straightforward and the previous results generalize in toto to this case even when there are transfers. In fact, the following is an easy corollary:

Corollary 2. *Propositions 2-4 hold for Case 1 in which transfers are fixed as fraction of GDP.*

For Case 2 matters are slightly more complicated. The reason is that in addition to the negative substitution effect of the stricter tax law enforcement there is an another effect that arises in the presence of transfers. A stricter enforcement increases government revenue, and in Case 2, a part of that increase is rebated to the agents. As these transfers are not taxed, the exogenous or non-taxable component of the income rises. Being a CRRA consumer, when the agent decides to invest a constant share of his total income (including both wage income and transfers) in the risky activity of

tax evasion, the income on which tax is evaded rises as a fraction of the wage income. This happens because the wage income becomes smaller as a fraction of total income when transfers increase.

Once again, in our simplified set up, this mechanism comes out clearly in the analytics of the model. To illustrate this effect, we begin by noting that while the agent evades taxes today, the resulting risk affects only future consumption: he has already consumed in the current period by the time he knows whether he has been caught evading taxes. Since, the risk pertains to future consumption, the relevant 'income' for this purpose is not the current wage income but the saving, which finances future consumption. In fact, with little algebra one can show that, as a fraction of saving $(\tilde{s}^*(1-\alpha)y^*)$, the income on which tax is evaded $(x^*\tau_i(1-\alpha)y^*)$ is given by

$$x_s^* \equiv \frac{x^*\tau_i}{\tilde{s}^*} = \frac{\frac{\tau_i}{\tau_i^p} - p}{1 - p}. \tag{29}$$

and is constant for a given government policy. Thus, any exogenous change in saving will be reflected in tax evasion. In our case, the relevant exogenous change in saving comes from the increased transfers received by the agent, a part of which is saved. The resulting increase in saving will tend to raise tax evasion.

While this precludes a generalization of the results of the previous section to all time-invariant policies of Case 2, the understanding of the mechanism allows us to make some progress. To do so first note that this effect is absent, and therefore the previous results hold, for $\zeta_R = 0$. Second, we can also see that this additional effect will become stronger as ζ_R increases: a higher ζ_R implies that a larger proportion of the increase in government revenue is rebated back to the agent. Thus, saving rises proportionately more for a higher value of ζ_R implying higher tax evasion as well. Therefore, we have the following corollary that applies to Case 2 with non-zero transfers:

Corollary 3. *Propositions 2-4 hold for Case 2 in which transfers are fixed as fraction of government revenue as long as $\zeta_R < \bar{\zeta}_R$, where $\bar{\zeta}_R$ is a function of government policy, $\{p, \tau_i^p, \tau_i\}$.*

4.3 Comparative Dynamics

The previous sub-section demonstrated that stricter tax law enforcement reduces tax evasion in the long run as long as the proportion of additional revenue generated that is rebated back to the agents is not very large. The reduced tax evasion was also

accompanied by reduced aggregate saving and private capital accumulation. While, what happens in the long run is no doubt important, many a times, what happens is the short run is far more important, especially if the continuation of policy is contingent on delivering immediate results.

In this section, we extend the results of comparative static exercises to the entire dynamic path of the economy. However, before we look at how the economy responds to a government policy change along the dynamic path (i.e. at the comparative dynamics), for the simplified model, it is useful to first analyze the (equilibrium) dynamics of the economy when government policy is time-invariant in an appropriate sense. As we shall see, the comparative dynamics follows easily from the equilibrium dynamics and the comparative statics, with latter determining the impact effect of the policy change and former determining the subsequent outcome.

4.3.1 Equilibrium Dynamics

The characterization of the equilibrium dynamics of the economy under time-invariant policies in this model can be conveniently split into two parts. The next proposition summarizes the behavior of key macroeconomic ratios and tax evasion. The behavior of the aggregate dynamics of the economy follows thereafter.

Proposition 5. *With time-invariant government policies as in either Case 1 or Case 2, the equilibrium dynamics of the economy is characterized by:*

1. a *constant level of tax evasion.*

2. a *constant ratio of stocks of the private and the public capital.*

3. *the constant ratios of private capital, public capital and ex ante savings to the output in every period.*

4. a *constant ratio of consumption of generation t to the output in period t.*

The fact that the ratio of the public and the private capital is constant along transition reveals the nature of the underlying aggregate dynamics. It implies that the dynamics of the economy can be described by one state variable. In fact, a little algebra shows that the equations describing the evolution of aggregate private capital stock, (22a) or (22b) as the case may be, has the form

$$K_{t+1} = bK_t^{\alpha+\theta} \quad \text{for some} \ \ b > 0 \ \ \text{and} \ \ 0 < \alpha + \theta < 1, \tag{30}$$

where b is a function of the time-invariant government policy. Thus, the dynamic behavior of this economy is like that of the ordinary neo-classical economy without tax evasion and public capital and shows monotonic convergence to the steady state. If the economy starts with the stocks of private and public capital below their steady state levels, these variables increase monotonically to their steady state values. More generally,

Corollary 4. *The stocks of both public and private capital, and consumption when young and in the two states when old, converge monotonically to their steady state values.*

4.3.2 Tax Evasion, Saving and Capital Accumulation

The previous section showed that, once a time-invariant policy is in place, the transition dynamics of the aggregate variables is characterized by monotone convergence to their new steady state values. While aggregate variables change during transition, tax evasion and saving, however, remain constant along the entire transition path.

These two observations allow us to assess the economy's response to a one time change in p or τ_i^p which constitutes a move to a new time-invariant policy. In particular, it easily follows that, while aggregate variable may change during transition, saving and tax evasion can change only on impact and cannot change thereafter during the transition. This in turn implies that the change in their values on impact must be the same as that across the steady states. Thus, in light of Proposition 5, with time-invariant government policy as defined in either Case 1 or Case 2, the comparative static results immediately extend to the entire dynamic path.

Corollary 5. *In Case 1, a one time increase in p or τ_i^p causes an immediate and permanent decrease in tax evasion and saving to their new steady state values. For Case 2, this happens for $\zeta_R < \bar{\zeta}_R$.*

It needs to emphasized that the economy does *not* immediately jump to its new steady state. Typically, the change in policy will change the level of aggregate variables and these variables will only converge to their steady state values over time. It just so happens that variables of primary interest–tax evasion and saving–do not show transitory dynamics. In fact, this is also true for the capital-output ratios as well:

Corollary 6. *In Case 1, one time increase in p or τ_i^p causes an immediate and permanent decrease in the private-capital-to-output ratio (K_{t+1}/Y_t) and an immediate and permanent increase in the public capital-to-output ratio (G_{t+1}/Y_t) to their new steady state values. For Case 2, this happens for $\zeta_R < \bar{\zeta}_R$.*

Corollary 5 generalizes the results in existing literature to the policy relevant case where government action is purposeful. The stricter enforcement of tax laws results in an increase in government revenue which is used productively and the effect of this expenditure on the economy is taken into account. Furthermore, it shows that the comparative static effects of policy change on tax evasion and saving materialize not just in the long run but immediately at the time of policy change.

While a useful result in itself, from a practical standpoint, one might argue that the comparative dynamics of the model is trivial. More generally, one might question the generality of the results. This is a valid concern, but, I think, the analysis of the simplified model serves a very important purpose. It analytically demonstrates that the extant results in the literature *can* hold in a stylized dynamic general equilibrium setting. Therefore, it serves as a very useful benchmark–while analytical characterization is not possible in a more general dynamic general equilibrium setting but knowledge of the results for benchmark case is a useful starting point for what to expect. The numerical analysis of the next section shows that the results for the more realistic models are not too different from that for this stylized model.

5   Numerical Simulation in a More General Model

The purpose of this section is to ascertain whether the results for the stylized model are special or they actually hold more generally. While simplifications of the previous section allowed analytical characterization of the effects of government policy on tax evasion and other macroeconomic variables, the assumption of log preferences, with coefficient of relative risk aversion of 1, was particularly restrictive. Since the effect of government policy on tax evasion is likely to critically depend on the degree of risk aversion, in numerical simulations we work with CRRA preferences

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma}.$$

With these preferences, it is possible to match the empirical evidence on the degree of risk aversion by choosing an appropriate value of σ, the coefficient of relative risk

aversion. These are also the most widely used preferences in the dynamic general equilibrium models.

As a result of these preferences, Corollaries 5 and 6 no will longer hold and, in particular, tax evasion and saving will vary along the transition because the dynamics of these variables cannot be separated from the dynamics of the aggregates. Furthermore, it also implies that the impact effect of the policy change will be different from the long-run effect; and hence, analysis of policy cannot be limited to comparative statics.

Besides the preferences, the general model also differs from simplified model in allowing for a partial depreciation of both types of capital in accordance with the empirical facts. This will impart additional transitional dynamics to the paths of aggregate capital stocks; and as preferences are no longer logarithmic, this will in turn impart further dynamics to the behavior of saving and tax evasion.

5.1 Calibration of the Model

The model is calibrated to match the characteristics of the developing economies. While calibrating the model, parameters have been assigned values that are reasonable for the developing countries, based on estimates in the existing literature. In addition, the values of β, x, and τ_i^p have been arrived at by calibrating the model to match the data on r, p, R/Y.

Durlauf and Johnson (1992) study the convergence across national economies. They find that the share of physical capital in the output/income varies between 0.30 and 0.40. The poor countries have a capital share of income in the output of 0.30, whereas for the countries with intermediate income it is 0.40. For the developed countries they find this to be 0.33. For Latin American economies, Elias (1992) estimates a value of 0.50. Hence, we choose $\alpha = 0.40$ which is in the middle of these estimates. There are very different estimates of the elasticity of national output with respect to public capital varying from close to zero to 0.20 (see Lynde and Richmond, 1993 and Ai and Cassou, 1995) and θ is given the middle value of 0.1.

Following Turnovsky (2004), annual depreciation rates of the private and the public capital are set at 5% and 3.5%. As each period in the model corresponds to 25 years, this yields $\delta_k = 0.723$ and $\delta_G = 0.59$. The tax rate on labor income is set to the typical value of 30% which was also the marginal tax rate for India and Pakistan for $2001 - 2002$ (also see Turnovsky, 2004).

It is quite easy to see that in this model with tax evasion, the risk aversion

Preferences	
$\beta = .1139;$	$\sigma = 2.0$
Production Function	
$\alpha = .4;$ $\theta = .1;$ $A = 100;$ $\delta_k = .723;$ $\delta_G = .64$	
Government Policy	
$\tau_i = .30;$ $\tau_i^p = .521;$ $\zeta = .85$	
Other	
$p = .15$	

Table 1: Parameter values for the calibrated model.

characteristics of the individuals is very important. The model is calibrated for $\sigma = 2$ which is close to the micro estimates obtained by Ogaki, Ostry, and Reinhart (1996) and Ogaki and Reinhart (1992).

As stated earlier, the values of β, x, and τ_i^p are chosen to match the data on r, p, R/Y. The ratio of government revenue to GDP can be ascertained from Summers and Heston (1992)/ Penn World Tables (Mark 5.6a) which reveals a considerable variation in this ratio. It varies from 10% to 30% for the middle 90% of the countries and the average is lower for the developed countries than for the developing countries. Since the model only has tax on labor income, whereas for developing countries tariffs are a significant source of revenues, the model is calibrated for $R/Y = 0.15$. The ratio of public investment to government revenue $(1 - \zeta)$ is set to match the ratio of public capital to private capital (G/K) of .368 in Turnovsky (2004). Setting $\zeta = .85$, yields $G/K = .3828$. This choice also matches the finding in Glomm and Rioja (2003) that 15% of the revenue is spent on infrastructure in Latin America.

The real interest rates again vary quite widely across countries; however, a 6% annual real interest rate is assumed which is on the lower side of the range of values for the developing countries. The probability of being caught, p, is fixed at 0.15 which implies that every year less than 1% of the returns are audited. This corresponds to the audit rate in developing countries such as India.

The values of parameters for the calibrated model are given in Table 1. The steady state of the calibrated model is summarized in Table 2. The annualized capital-output ratio is in the middle of the range of values for the developing countries (see Buffie, 2001, chapter 5). The consumption age profile has a slightly downward slope in

Tax Evasion
$x = .2254$
Consumption-Output Ratios
$c_y/y = .4779;$ $c_o/y = .4276;$ $c_o^1/y = .4730;$ $c_o^2/y = .1705$
Savings/*Capital-Output Ratios*
$s/y = .1102;$ $k/y = .0996;$ $k_1/y = .1102;$ $k_2/y = .0397$
Annualized $k/y = 2.4907$
Government
$R/y = .15;$ $J/y = .1275;$ $G/y = .0381;$ $G/K = .3828$
Other
$r = .06;$ $Y = 822.20$

Table 2: Steady state for the calibrated model.

accordance with the empirical observations. Agents evade tax on 22.54% of their income which is on the lower side of the estimates. Overall the calibrated model captures important features of the developing countries.

5.2   Comparative Dynamics of the Calibrated Model

In this section, we present the results of numerical simulations for Case 2 which is the case in which the results in the existing literature are less likely to hold in our setup. Figure 1 show the transition dynamics of the four ratios–tax evasion, saving, K/Y, G/Y–when the economy starts from a position where its stocks of public and private capital are 60% of their steady state levels. It also shows the new transition path and the new steady state values of these ratios resulting from a 10% increase in p.

Beginning with the transition dynamics, note that as expected, in the general model, tax evasion and saving vary over time as they now depend on the aggregate state of the economy. As the aggregate capital stock rises over time, the return on capital falls. With inter-temporal elasticity of substitution of $1/\sigma = .5$, the agents have a stronger incentive to smooth consumption than the log-case which causes individual saving to rise as fraction of income which, in turn, raises aggregate saving-to-output ratio. This rise in saving, as fraction of income, increases tax evasion
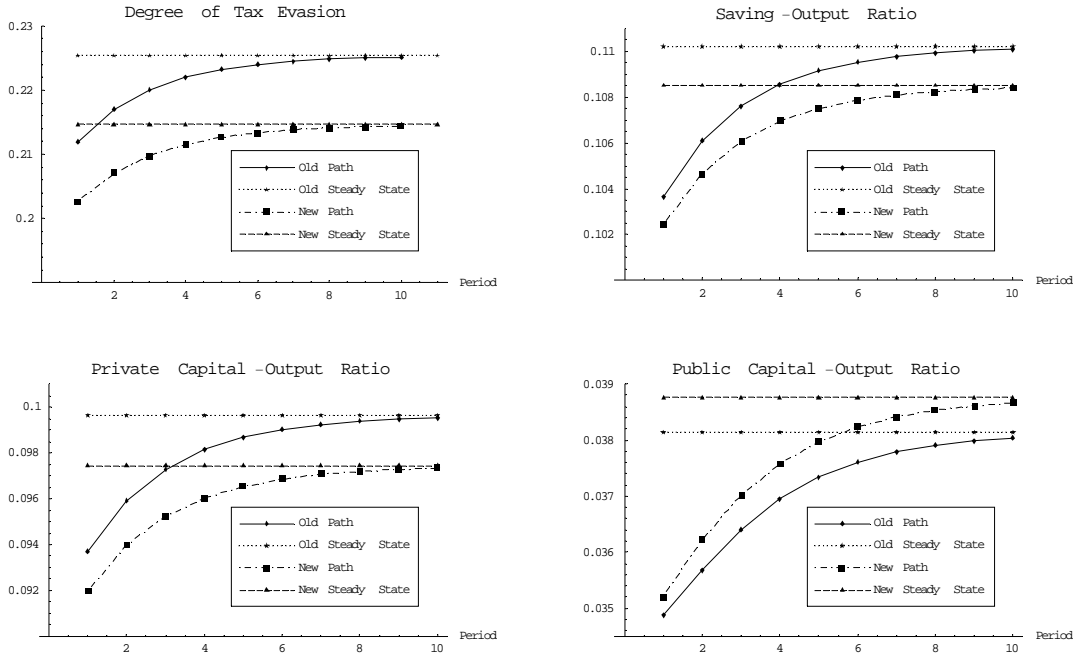
Figure 1: Comparative dynamics of the calibrated model for an increase in p.

through the mechanism described earlier, when discussing the case with transfers for the simplified model. While this increase in saving increases the private-capital-to-output ratio to allow the agent to smooth consumption, a part also goes to increase government revenue which raises the public-capital-to-output ratio.

A look at Figure 1 reveals that the comparative dynamics of the general model is very similar to that of the simplified model in some important respects. As in the simplified model, an increase in p decreases tax evasion and saving immediately and along the entire future equilibrium path. Similarly, private-capital-to-output ratio falls whereas public-capital-to-output ratio rises. From the point of view of policy-makers, it is important that the tax evasion declines immediately rather than just the across steady states. The impact effect is slightly smaller than in the long run, but not by very much. For example, on impact tax evasion falls by .935 percentage points, whereas, across steady states, the decrease is slightly higher at 1.075 percentage points. The difference between the impact and the steady state outcome is slightly larger for macroeconomic ratios–for example, 1.208 versus 1.697 percentage points for saving.
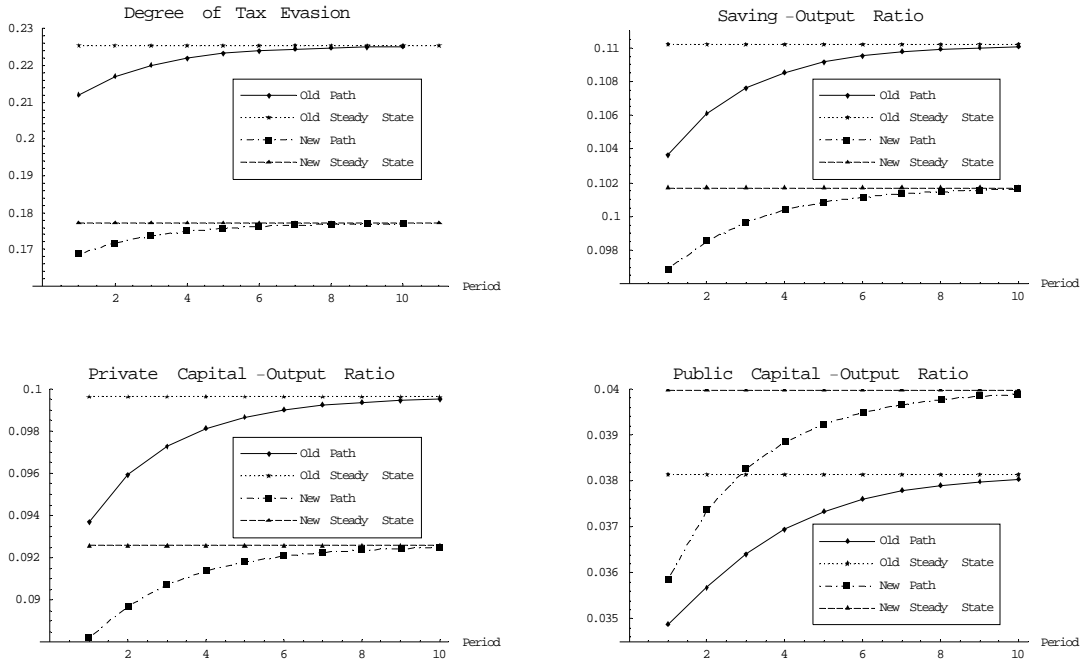
Figure 2: Comparative dynamics of the calibrated model for an increase in τ_i^p.

The comparative dynamics for an 10% increase in τ_i^p is shown in Figure 2. The outcome for the general model is again similar to the simplified model as was the case for an increase in p. Therefore, the earlier discussion applies.

While we undertook an elaborate calibration the model to match the characteristics of the developing economies, the comparative dynamics of the model is *very robust* to the parameter values.[14] In addition, as the results from numerical simulations presented above are arrived at by solving the actual non-linear model, they do not have any linearization or approximation errors.[15] Thus, the transition paths shown in Figure $1 - 2$ are the actual paths of the relevant variables.

6   Conclusion

While tax policy changes in practice are purposeful and aim at raising government revenue for productive uses, current analysis of tax evasion ignores government's expenditure policy. In this paper, we have examined the effect of policy changes on tax evasion for such purposeful changes in policy that increase public investment

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[14]We did not discover any parameter combination for which the results for stricter enforcement were different from those described above.

[15]Given the precision of numerical algorithms, errors from numerical estimation are insignificant.

in infrastructure. We show that the standard comparative static results for stricter tax law enforcement hold if the fraction of additional revenue generated (from tax policy) that is rebated to the agents is not too large. Furthermore, although the direction of the effect on tax evasion during the transition is same as that across the steady states, the short-run responses are slightly smaller than the long-run response. These results are useful from policy perspective as they show that an increase in the audit rate or the penal tax rate causes an *immediate* and lasting decrease in tax evasion rather than just across steady states. The stricter enforcement also has important macroeconomic implications. Both saving and private capital accumulation fall along with tax evasion. This negative effect counteracts the positive effect of stricter enforcement on growth that comes from a greater investment in public capital made possible by increased revenue collection.

The paper provides a tractable framework for studying issues for which tax evasion may be an important consideration. For example, Atolia (2009) employs it to evaluate the welfare effect of tariff reforms in developing countries with rampant tax evasion. For future work, an extension to flexible labor supply will be useful as the agents will now have another margin to 'self-insure' when deciding to evade taxes.

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